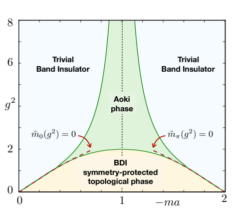
Correlated Chern insulators in two-dimensional Raman lattices

a cold-atom regularization of strongly-coupled four-Fermi field theories

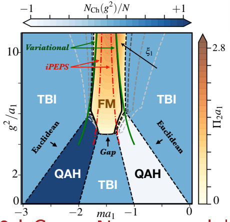
Emanuele Tirrito

SISSA

A. Bermudez, E. Tirrito, M. Rizzi, M. Lewenstein, S. Hands, Annals of Physics 399, 149 (2018)
L. Ziegler, E. Tirrito, M. Lewenstein, S. Hands, A. Bermudez, arXiv:2011.08744 (2020)
L. Ziegler, E. Tirrito, M. Lewenstein, S. Hands, A. Bermudez, arXiv:2111.04485 (2021)



1d Gross-Neveu model

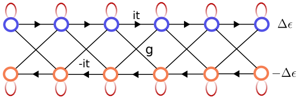


2d Gross-Neveu model

Gross-Neveu model

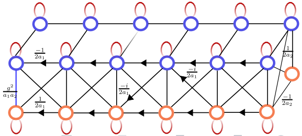
Atomic Physics

Synthetic Quantum Matter



Condensed Matter Physics

topological states of matter



Gross-Neveu Model

$(1+1)D$ toy model for QCD

D. J. Gross and A. Neveu PRD 10, 3235 (1974)

$$\mathcal{H} = -\overline{\Psi}(x) i\gamma^1 \partial_x \Psi(x) - \frac{g^2}{2N}\left(\overline{\Psi}\Psi\right)^2$$

$g^2 > 0$ → IR relevant → Spontaneus breaking of (discrete) chiral symmetry → dynamical mass generation

$g^2 > 0$ → UV vanishing → Asymptotic freedom

The Gross-Neveu model on the lattice

$$\mathcal{H} = -\overline{\Psi}(x) i\frac{\gamma^1}{2a}\Psi(x+a) - \frac{g^2}{2N}\left(\overline{\Psi}\Psi\right)^2 + m\overline{\Psi}(x)\Psi(x) + \mathcal{H}_W$$

Where $\mathcal{H}_W$ is the Wilson term

$$\mathcal{H}_W = \overline{\Psi}(x)\frac{r}{2a}\left(\Psi(x) - \Psi(x+a) + H.c.\right)$$

Critical line $m_c(g^2)$? Topological invariant (Berry's phase)?

Non-interacting limit of the GNW model

Wilson lattice discretization to avoid doublers!

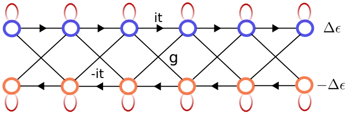
J. Kogut, L. Susskind PRD 11, 395 (1975)

K. Wilson, New Phenomena in Subnuclear Physics

H. B. Nielsen, M. Ninomiya Nuc. Phys B (1981)

$$h_k = \left(m + \frac{1-\cos(ka)}{a}\right)\gamma^0 - \frac{\sin(ka)}{a}\gamma^5$$

is essentially a Creutz-Ladder



$$h_k^{CL} = \left(\frac{\Delta\epsilon}{2} + 2t\sin(ka)\right)\gamma^0 - 2t\cos(ka)\gamma^5$$

$$ma = \frac{\Delta\epsilon}{4t} \qquad g^2 = \frac{V_v}{2t} \qquad (N=1)$$

By diagonalization $\longrightarrow$ $\epsilon_\pm(k) = \pm\frac{1}{a}\sqrt{(ma+1-\cos ka)^2 + \sin^2 ka}$

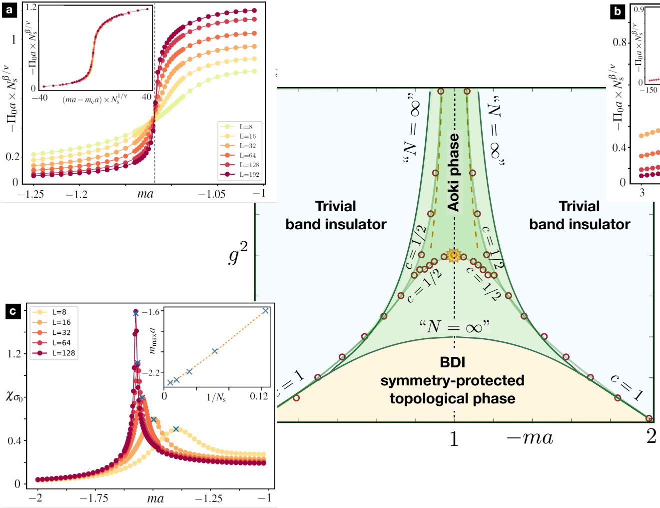
Berry Connection

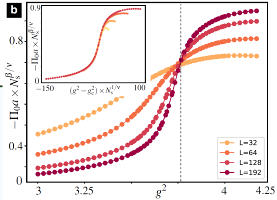
$$\mathcal{A}_n(k) = \frac{1}{2}\frac{(1+ma)\cos ka - 1}{1+(1+ma)^2 - 2(1+ma)\cos ka}$$

Zak's Phase

$$\varphi_{\text{Zak}} = \frac{1}{2}N\pi\left(\text{sgn}(\tilde{m}_\pi) - \text{sgn}(\tilde{m}_0)\right)$$

Phase diagram





Pseudoscalar Condensate

$$\Pi_0 = \langle \bar{\psi} i \gamma^5 \psi \rangle = -\frac{2}{a} \langle T_i^y \rangle$$

Susceptibility

$$\chi = \frac{\partial \sigma_0}{\partial m}$$

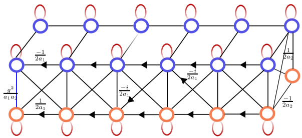
with $\sigma_0 = \langle \bar{\psi} i \gamma^0 \psi \rangle$

Gross-Neveu Model in $2+1$ dimension

$(2+1)D$ **toy model for QCD**

$$\mathcal{H} = -\overline{\Psi}(x)\left(i\gamma^1\partial_x + \gamma^2\partial_y\right)\Psi(x) - \frac{g^2}{2N}\left(\overline{\Psi}\Psi\right)^2$$

Wilson-type lattice fields theories



$$H = a_1 a_2 \sum_x \sum_{j=1}^{2} \Bigg(-\overline{\Psi}(x)\left(\frac{i\gamma^j}{2a_j} + \frac{r_j}{2a_j}\right)\Psi(x + a_j e_j)$$
$$+\overline{\Psi}(x)\left(\frac{m}{4} + \frac{r_j}{2a_j}\right)\overline{\Psi}(x) + H.c.\Bigg) - \frac{g^2}{2N}\left(\overline{\Psi}(x)\overline{\Psi}(x)\right)^2$$

Non interacting regime $\Rightarrow$

$$h_k(m) = \sigma^x \sin(k_1) + \sigma^y \sin(k_2) + (m + 2 - \cos(k_1 a_1) - \cos(k_2 a_2))\,\sigma^z$$

Chern number	Conductivity

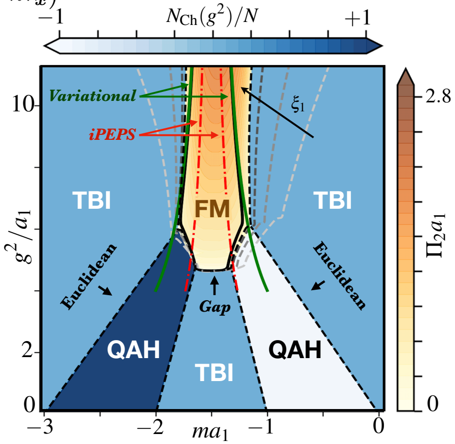
$$N_{ch} = \frac{N}{2}\sum_{nd}(-1)^{n_{d,1}+n_{d,2}} sign(m_{n_d})$$

$$\sigma_{xy} = \frac{e^2}{2\pi}\sum_n N_{Ch,n}$$

Gross-Neveu Model in $2+1$ dimension

Strong Interactions → **Compass Model**

$$H_{\text{eff}} = \sum \left(J_x \tau^x_{\boldsymbol{x}} \tau^x_{\boldsymbol{x}+a_2\mathbf{e}_2} + J_y \tau^y_{\boldsymbol{x}} \tau^y_{\boldsymbol{x}+a_1\mathbf{e}_1} - h\tau^z_{\boldsymbol{x}} \right)$$



Gap equations

$$\frac{\partial S_E}{\partial \sigma} = 0 \quad \frac{\partial S_E}{\partial \Pi_1} = 0 \quad \frac{\partial S_E}{\partial \Pi_2} = 0$$

Condensates

$$\sigma = \langle \overline{\Psi}(x)\Psi(x) \rangle \quad \Pi_1 \propto \langle \overline{\Psi}(x)\gamma^1\Psi(x) \rangle$$

$$\Pi_2 \propto \langle \overline{\Psi}(x)\gamma^2\Psi(x) \rangle$$