Calibration of Electromagnetic Fields in Three Dimensions by Precision Spectroscopy of Hydrogen Molecular Ions

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Introduction

Sensitive detection of the electromagnetic fields has a broad range of fundamental and technological applications including tests of the fundamental symmetries, timekeeping, astrophysical observation, medical imaging, geophysics and wireless communications. The microwave electric field measurements based on Rydberg atom spectroscopy enabled sub-wavelength resolution, accuracy, long-term reproducible operation, SI traceability with weak electric field detection limit at the µV/cm level imposed by the photon shot noise [1,2].

This contribution proposes to measure the THz electric fields using precision spectroscopy with cold trapped HD^+ ions [3]. Calibration with traceability to the SI second and to the fundamental constants may be performed by comparing the measurements of the systematic frequency shifts induced on HD^+ lines with the predictions of the molecular ion theory.

Ab-initio HD^+ energy levels and shifts in external fields

$$E(v,L,F,S,J,J_z)=E_{rv}(v,L)+E_{hf}(v,L,F,S,J)+\Delta E_Z(v,L,F,S,J,J_z;B)+\Delta E_{LS}(v,L,F,S,J,J_z;q,B,f_{THz})$$

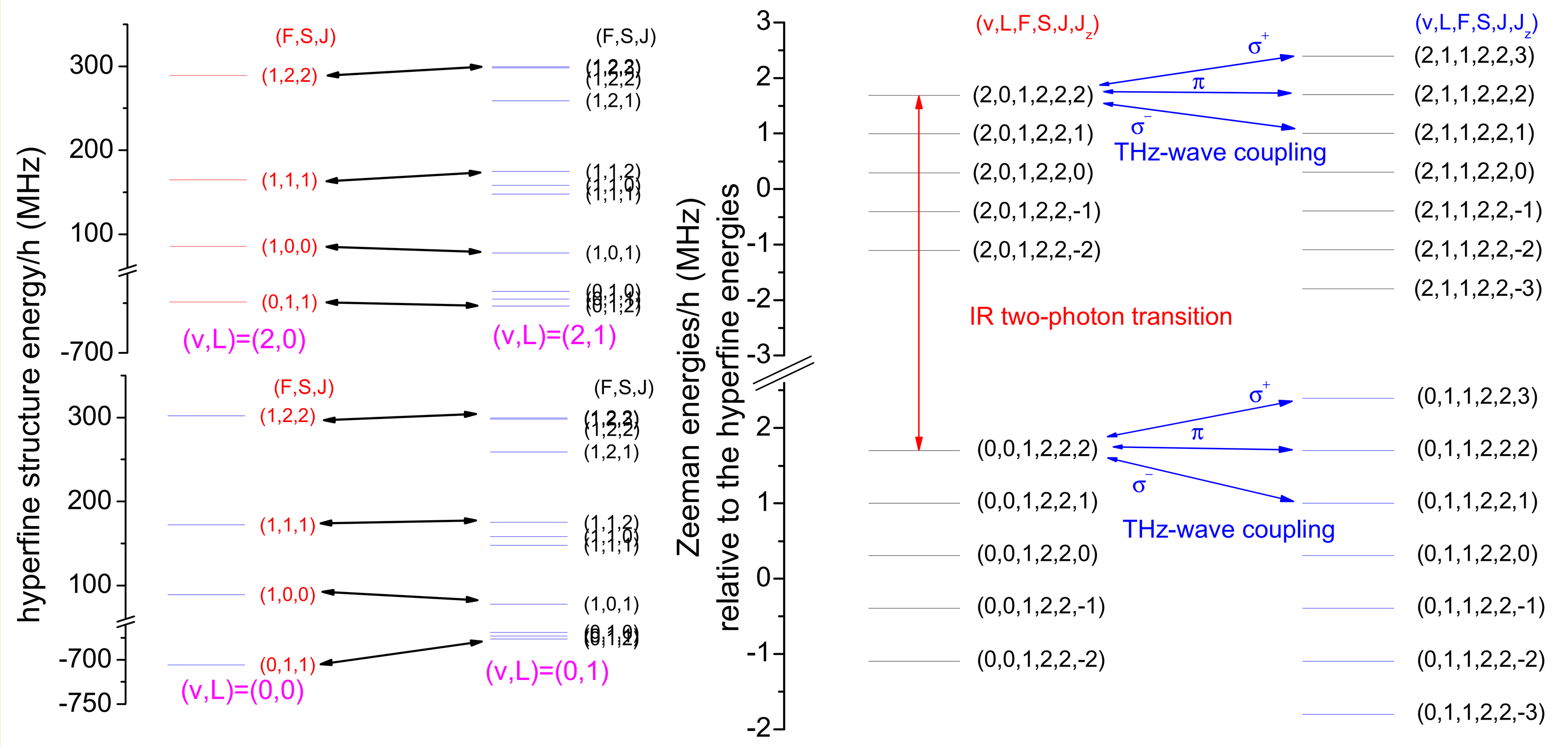
-rovibrational energies: 10^{-12}-level precision [4]

-hyperfine splittings: 0.5 kHz accuracy [5]

-magnetic level shift by Zeeman effect [6]:

$$\Delta E_Z(\{U_{th}\};B)\approx h\left[tJ_zB+\left(q+rJ_z^2\right)B^2\right]$$

10^{-4}-level precision for t,q,r parameters

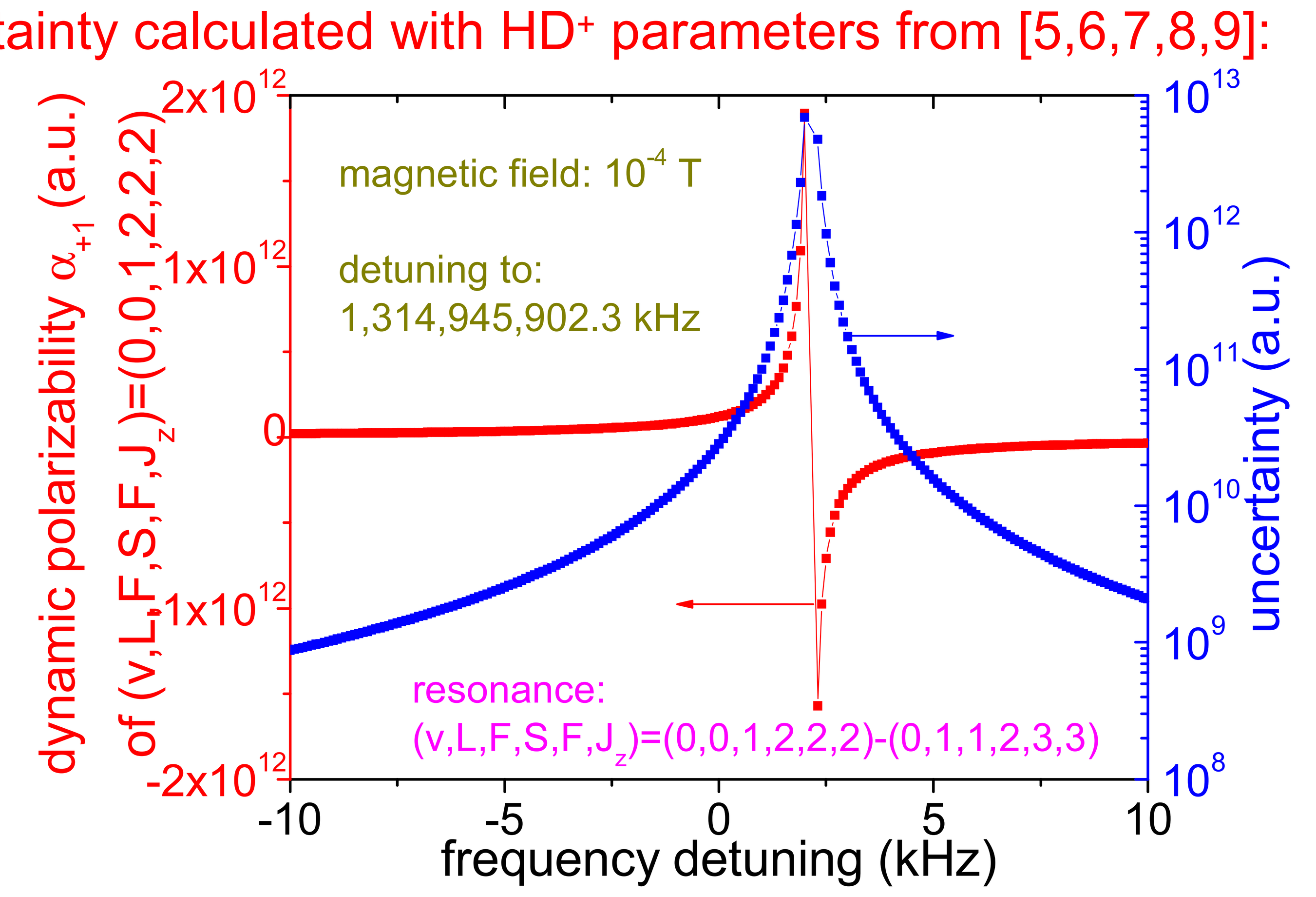


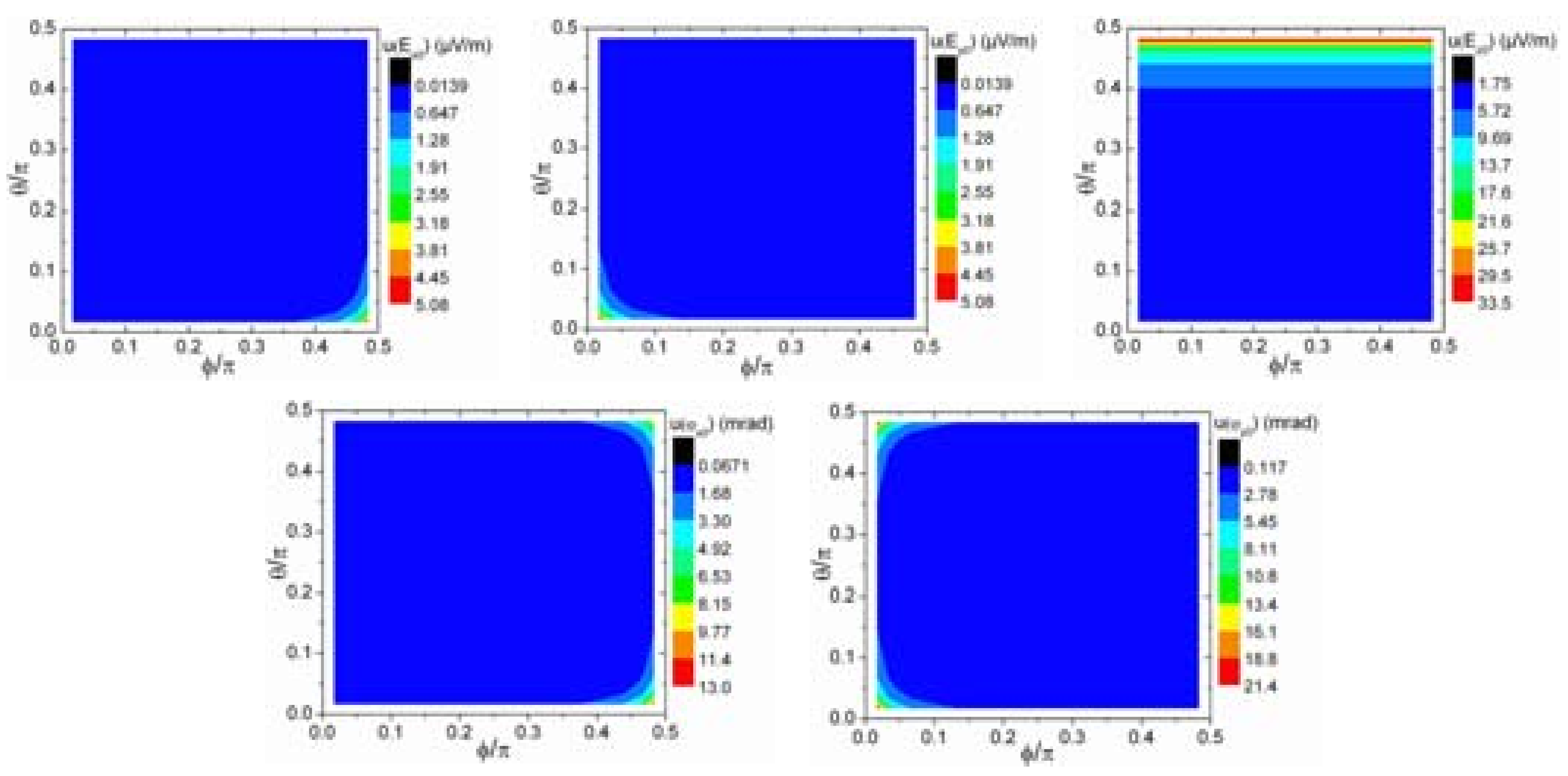
Lightshifts quantified with the standard components of the THz electric field and the *standard dynamic polarizabilities* of HD^+:

$$\Delta E_{LS}(n;|E_{THz}|)=-\frac{1}{4}|E_{THz}|^2\sum_r Re\left[\frac{\langle n|(\vec d\cdot\hat\varepsilon)^+|r\rangle\langle r|(\vec d\cdot\hat\varepsilon)|n\rangle}{E_r-E_n-\hbar\omega-i\hbar\frac{\gamma_r+\gamma_n}{2}}+\frac{\langle n|(\vec d\cdot\hat\varepsilon)|r\rangle\langle r|(\vec d\cdot\hat\varepsilon)^+|n\rangle}{E_r-E_n+\hbar\omega+i\hbar\frac{\gamma_r+\gamma_n}{2}}\right]$$

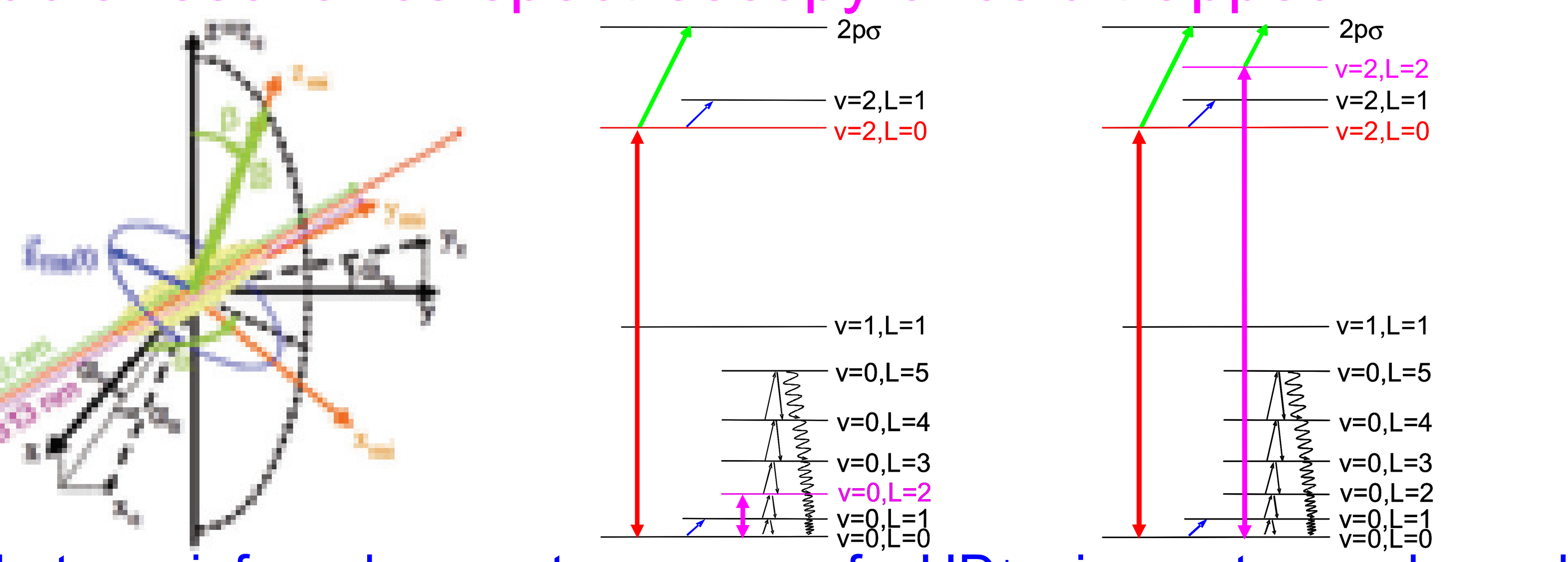
$$\Delta E_{LS}(n;E_{THz,-1},E_{THz,0},E_{THz,1},B,f_{THz})=-\frac{1}{4}\sum_{q=\{-1,0,1\}}(-1)^q|E_{THz,-q}|^2\alpha_{n,q}(\{U_{th}\};q,B,f_{THz})$$

Frequency dependence of a standard dynamic polarizability and its uncertainty calculated with HD^+ parameters from [5,6,7,8,9]:





Double resonance spectroscopy of cold trapped HD^+



-Two-photon infrared spectroscopy of HD^+ ions trapped and sympathetically cooled with 313 nm laser-cooled Be^+ ions

-Detection by dissociation of (v,L)=(2,0) with 175-nm laser radiation

=>Limit of the fractional resolution beyond the 10^{-12} level [10]

=>Frequency measurements with QPN-limited uncertainty: rovibrational transition at ~2 Hz; rotational transition at ~71 mHz [10]

-Relate measurements in a slightly nonorthogonal reference frame *CCF*(x_c,y_c,z_c) defined with three coil pairs to the ideal Cartesian laboratory frame *LCF*(x,y,z) using an adjustable frame *MICF*(x_{mi},y_{mi},z_{mi})

1) Zeeman spectroscopy on (v,L)=(0,0)→(2,2) @ 57.701 THz or (v,L)=(0,0)→(0,2) @ 1.968 THz

=>Magnetic field strength, orientation and the offset magnetic field

2) Lightshifts induced on (v,L)=(0,0)→(2,0) @ 55.909 THz by a THz-wave off-resonantly coupled to (v,L)=(0,0)→(0,1) @ 1.315 THz

=>Amplitudes and phases of the Cartesian components of the THz electric field

Calibration of a static magnetic field

-Relate spectroscopy measurements to a 3D static magnetic field:

$$\|\vec B_i\|=\frac{f_{exp,i}-f_{th,i}}{\eta_i}\Leftrightarrow\begin{cases}\vec B(\{V_k\};I_{1,i},I_{2,i},I_{3,i})=(k_1I_1-k_2I_2\alpha_z+k_3I_3\alpha_y+B_{01})\vec e_x\\ \quad+(k_2I_2-k_3I_3\alpha_x+B_{02})\vec e_y+(k_3I_3+B_{03})\vec e_z\end{cases}$$

9 experimental parameters: $\{V_k\}_i=\{k_1,k_2,k_3,\alpha_x,\alpha_y,\alpha_z,B_{01},B_{02},B_{03}\}$

3 theory parameters: $f_{th,i}=f_{rot,i}+f_{hf,i},\eta_i$

-Calibration of the experimental parameters using 27 measurements:

$$\Delta(k_1,k_2,k_3)=(10^{-12},10^{-11},10^{-11})\ \text{T/A};$$

$$\Delta(\alpha_x,\alpha_y,\alpha_z)=(10^{-7},10^{-7},10^{-8})\ \text{rad};$$

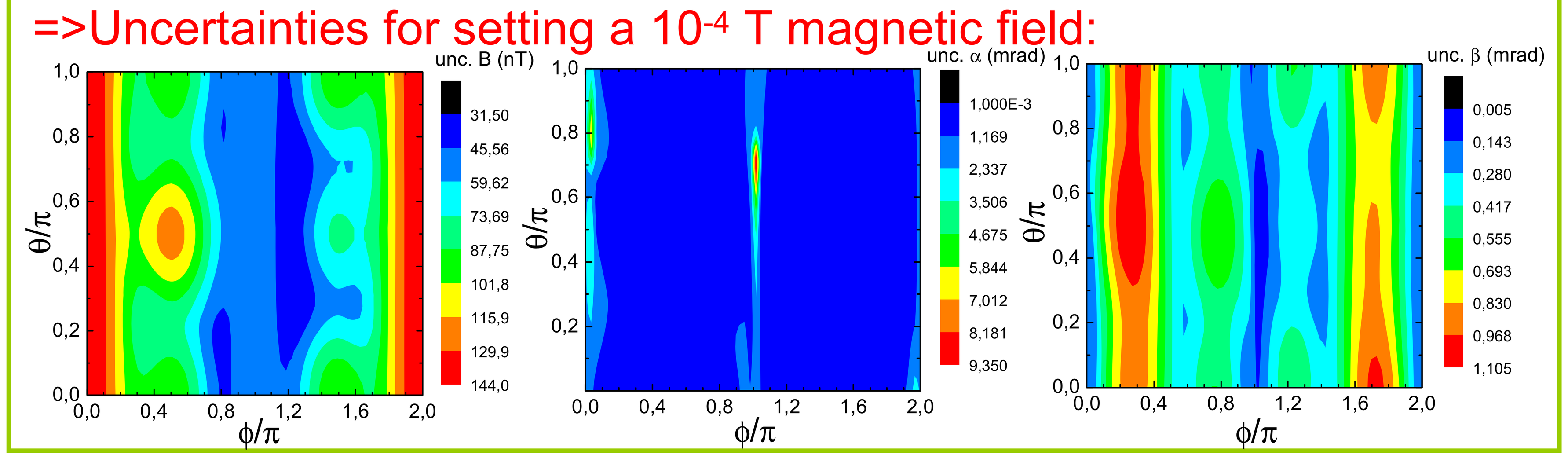
$$\Delta(B_{01},B_{02},B_{03})=(10^{-11},10^{-11},10^{-11})\ \text{T};$$

=>Cancellation at 85 nT of Earth's magnetic field with (I_{null1}, I_{null2}, I_{null3}):

-Current spherical coordinate parametrization: ($I_0\sin\theta\cos\phi+I_{null1}$, $I_0\sin\theta\cos\phi+I_{null2}$, $I_0\cos\theta+I_{null3}$)

-Magnetic field spherical coordinate parametrization: ($B(I_0,\theta,\phi)$, $\alpha(I_0,\theta,\phi)$, $\beta(I_0,\theta,\phi)$)

=>Uncertainties for setting a 10^{-4} T magnetic field:



3D characterization of a THz electric field

THz electric field in the *LCF*: $\vec E_{THz}(t)=\sum_{j=\{x,y,z\}}\frac{E_j\vec e_j}{2}\times e^{-i(\omega t+\varphi_j)}+c.c.$

Probe six lightshifts for two orientations and three values of the magnetic field on the (0,0,1,2,2,2)→(2,0,1,2,2,2) two-photon transition:

$$\delta f_i(I_{1,i},I_{2,i},I_{3,i})=-\frac{1}{4}\sum_{q=\{-1,0,1\}}(-1)^q\left|E_{THz,q}^{(\alpha_i,\beta_i)}\right|^2\Delta\alpha_{i,-q}\left(\{U_j\},\left\|\vec B(\{V_k\},I_{1,i},I_{2,i},I_{3,i})\right\|,f_{THz}\right)$$

Transformation relations: $\left|E_{\sigma^-,\pi,\sigma^+}^{(\alpha,\beta)}\right|^2=f_{\sigma^-,\pi,\sigma^+}(\alpha,\beta,E_x,E_y,E_z,\varphi_x,\varphi_y)$

Inversion of a nonsinglar system of equations:

=>6 standard components of the THz electric field in the *MICF* frame

<=5 parameters of linearily polarized THz electric field in the *LCF* frame

<=frequency 1,314,947,502.3 kHz, intensity 1 W/m^2

<=using calibrated values of the parameters $\{k_1,k_2,k_3,\alpha_x,\alpha_y,\alpha_z,B_{01},B_{02},B_{03}\}$

<=angles $(\alpha,\beta)=(0,\pi/2),(\pi/2,\pi/2)$ magnetic fields 10^{-6} T, 5×10^{-6} T, 10^{-5} T

<=level of uncertainty $\Delta(E_x,E_y,E_z)=(10^{-7},10^{-7},10^{-6})$ V/m; $\Delta(\varphi_x,\varphi_y)=(10^{-3},10^{-3})$ rad

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