Introduction

Sensitive detection of the electromagnetic fields has a broad range of fundamental and technological applications including tests of the fundamental symmetries, timekeeping, astrophysical observation, medical imaging, geophysics and wireless communications. The microwave electric field measurements based on Rydberg atom spectroscopy enabled sub-wavelength resolution, accuracy, long-term reproducible operation, SI traceability with weak electric field detection limit at the μV/cm level imposed by the photon shot noise [1,2]. This contribution proposes to measure the THz electric fields using precision spectroscopy with cold trapped HD⁺ ions [3]. Calibration with traceability to the SI second and to the fundamental constants may be performed by comparing the measurements of the systematic frequency shifts induced on HD⁺ lines with the predictions of the molecular ion theory.

Ab-initio HD⁺ energy levels and shifts in external fields

\[ E_{\text{microwave}}(v,F,J) = E_{\text{microwave}}(v,F,J) + \Delta E_{\text{f,B,q}}(v,F,J) + \Delta E_{\text{f,B,q}}(v,F,J) \]

-rovibrational energies:
-10⁻²³-level precision [4]

-hyperfine splittings:
-0.5 kHz accuracy [5]

Lightshifts quantified with the standard components of the THz electric field and the standard dynamic polarizabilities of HD⁺:

\[ \Delta E_{\text{f,B,q}}(v,F,J) = \frac{1}{4} \langle E_{\text{microwave}}(v,F,J) \rangle \sum_{\text{magnetic levels}} \left[ (\langle \Delta \tilde{d} \rangle \langle \Delta \tilde{d} \rangle^* \langle \Delta \tilde{d} \rangle^* \langle \Delta \tilde{d} \rangle) \right] \]

Frequency dependence of a standard dynamic polarizability and its uncertainty calculated with HD⁺ parameters from [6,7,8,9]:

Double resonance spectroscopy of cold trapped HD⁺

-Double resonance spectroscopy
-ION: cold trapped HD⁺

Calibration of a static magnetic field

-Relate spectroscopy measurements to a 3D static magnetic field:

\[ B = B(0,0,0) + B(1,0,0) + B(1,1,1) \]

9 experimental parameters:

9 \[ \alpha_i, \beta_i, \gamma_i \]

3 theory parameters:

3 \[ \alpha_{th}, \beta_{th}, \gamma_{th} \]

 Calibration of the experimental parameters using 27 measurements:

\[ \Delta B(0,0,0) \]

\[ \Delta B(1,0,0) \]

\[ \Delta B(1,1,1) \]

Cancellation at 85 mT of Earth’s magnetic field with \[ B_{\text{int}}, B_{\text{int}}, B_{\text{int}} \]

Current spherical coordinate parametrization:

\[ (\sin \psi \cos \theta, \sin \psi \sin \theta, \cos \psi) \]

Magnetic field spherical coordinate parametrization:

\[ (B_r, B_{\theta}, B_{\phi}) \]

Uncertainties for setting a 10⁻³ T magnetic field:

3D characterization of a THz electric field

THz electric field in the LCF:

\[ E_{\text{LCF}}(v) = \sum_{j=1}^{2} E_{j}(x,y,z) \]

Probe six lightshifts for two orientations and three values of the magnetic field on the (0,0,1,2,2,2)→(2,1,1,2,2,2) two-photon transition:

\[ \delta f \sim \sum_{(i,j)} \left[ (\Delta \tilde{d}_{ij})^2 \right] \]

Transformation relations:

\[ \delta \tilde{f}_j(x,y,z) = f_{\text{th}}(x,y,z) \]

Inversion of a nonsingular system of equations:

\[ \sum_{j=1}^{2} E_{j}(x,y,z) \]

References: