Arnoldi-Lindblad time evolution: Faster-than-the-clock algorithm for the spectrum of (Floquet) open quantum systems

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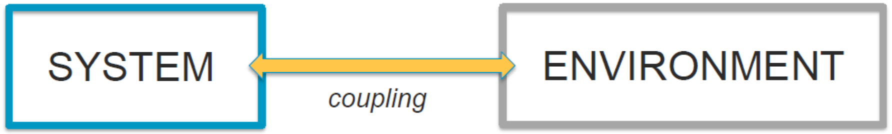


SYSTEM

coupling

ENVIRONMENT

Lindblad master equation

dissipation rate

Lindblad operator

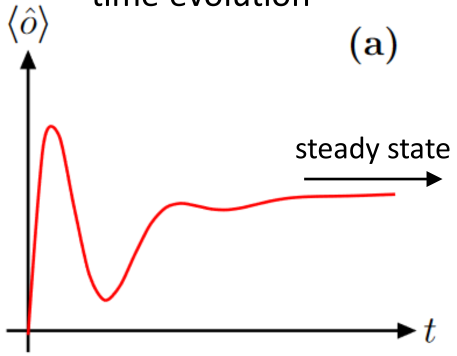
“dissipation operator”

$$\partial_t\hat{\rho} = \underbrace{-i[\hat{H},\hat{\rho}]}_{\text{closed system dynamics}} + \underbrace{\sum_j \frac{\gamma_j}{2}\left(2\hat{L}_j\hat{\rho}\hat{L}_j^\dagger - \{\hat{L}_j^\dagger\hat{L}_j,\hat{\rho}\}\right)}_{\text{coupling with environment}} = \mathcal{L}[\hat{\rho}]$$

$$\mathcal{L}\hat{\rho}_j = \lambda_j\hat{\rho}_j$$

Liouvillian superoperator

time evolution

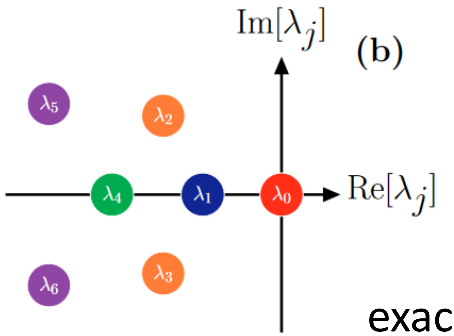


main interest in “slow” states e.g. the steady state

$$\hat{\rho}(t) = e^{\mathcal{L}t}\hat{\rho}(0)$$

$$= \sum_j c_j\, e^{\lambda_j t}\hat{\rho}_j$$

Liouvillian spectrum



Liouvillian eigendecomposition highly desired

exact diagonalisation often infeasible while time evolution is still accessible

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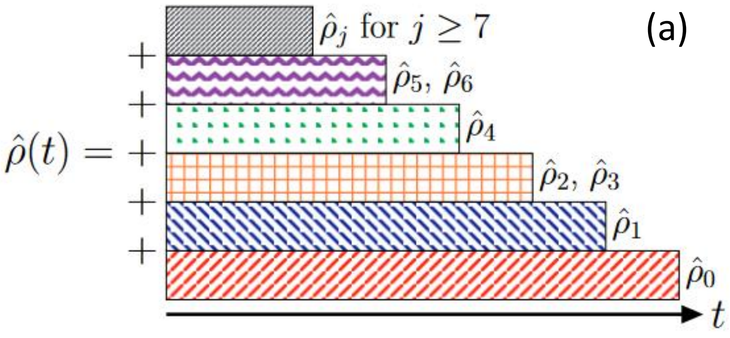
F. Minganti and DH, arXiv:2109.01648v2

THE METHOD

Time evolution

$$\hat{\rho}(t) = \hat{\rho}_{SS} + \sum_{j\geq 1} c_j e^{\lambda_j t}\hat{\rho}_j$$

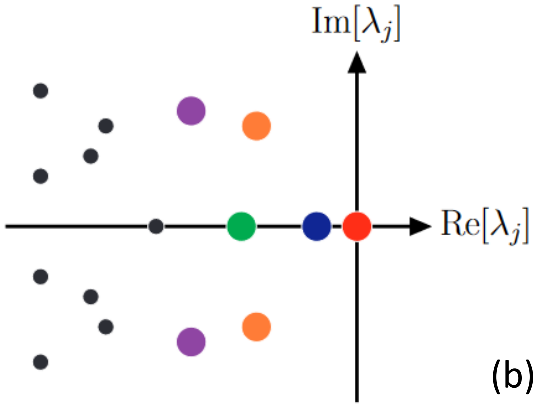


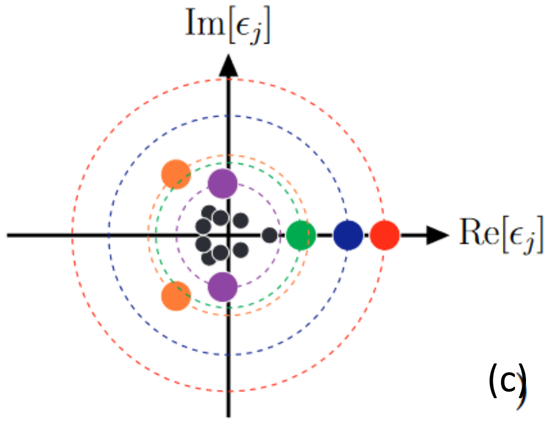
define a time step T

$$\mathcal{E} = e^{\mathcal{L}T}$$

$$\hat{\rho}(t) = \left[e^{\mathcal{L}T}\right]^N \hat{\rho}(0) = \mathcal{E}^N \hat{\rho}(0)$$

Spectrum transformation





$$\mathcal{E}\hat{\rho}_j = \epsilon_j\hat{\rho}_j = e^{\lambda_j T}\hat{\rho}_j \Leftrightarrow \mathcal{L}\hat{\rho}_j = \lambda_j\hat{\rho}_j$$

Krylov and Arnoldi method

$$\begin{aligned} K_n &= \{\hat{\rho}(0),\, \hat{\rho}(T),\, \hat{\rho}(2T),\, \ldots\, \hat{\rho}(nT)\} \\ &= \left\{\hat{\rho}(0),\, \mathcal{E}\hat{\rho}(0),\, \mathcal{E}^2\hat{\rho}(0)\, \ldots,\, \mathcal{E}^n\hat{\rho}(0)\right\} \end{aligned}$$

Arnoldi iteration

Gram-Schmidt orthonormalisation

$$\mathcal{E}_n^{\mathrm{eff}} = \begin{bmatrix} e_{1,1} & e_{1,2} & e_{1,3} & \cdots & e_{1,n} \\ e_{2,1} & e_{2,2} & e_{2,3} & \cdots & e_{2,n} \\ 0 & e_{3,2} & e_{3,3} & \cdots & e_{3,n} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & e_{n,n-1} & e_{n,n} \end{bmatrix}$$

check convergence

$$\left\|\mathcal{E}\hat{\rho}_j^{eff} - \epsilon_j^{eff}\hat{\rho}_j^{eff}\right\| < \tau$$

$$\mathcal{L}\hat{\rho}_j^{eff} = \boxed{\lambda_j^{eff}}\,\hat{\rho}_j^{eff}$$

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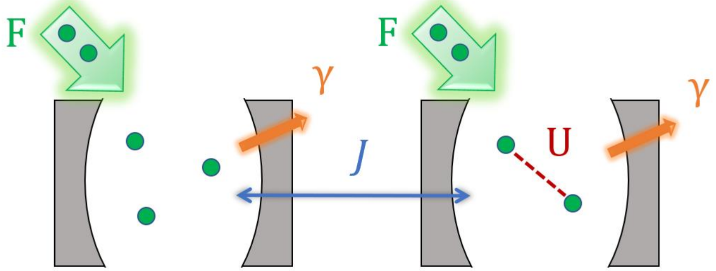
THE DRIVEN DISSIPATIVE BOSE HUBBARD DIMER

$$\hat{H} = \sum_{j=1}^{2}\left[-\Delta\hat{a}_j^\dagger\hat{a}_j + \frac{U}{2}\hat{a}_j^\dagger\hat{a}_j^\dagger\hat{a}_j\hat{a}_j + F\left(\hat{a}_j^\dagger + \hat{a}_j\right)\right]$$
$$-J\left(\hat{a}_1^\dagger\hat{a}_2 + \hat{a}_2^\dagger\hat{a}_1\right)$$



$$\partial_t\hat{\rho} = -i[\hat{H},\hat{\rho}] + \frac{\gamma}{2}\sum_{j=1}^{2}\left(2\hat{a}_j\hat{\rho}\hat{a}_j^\dagger - \{\hat{a}_j^\dagger\hat{a}_j,\hat{\rho}\}\right)$$

<u>parameters:</u>

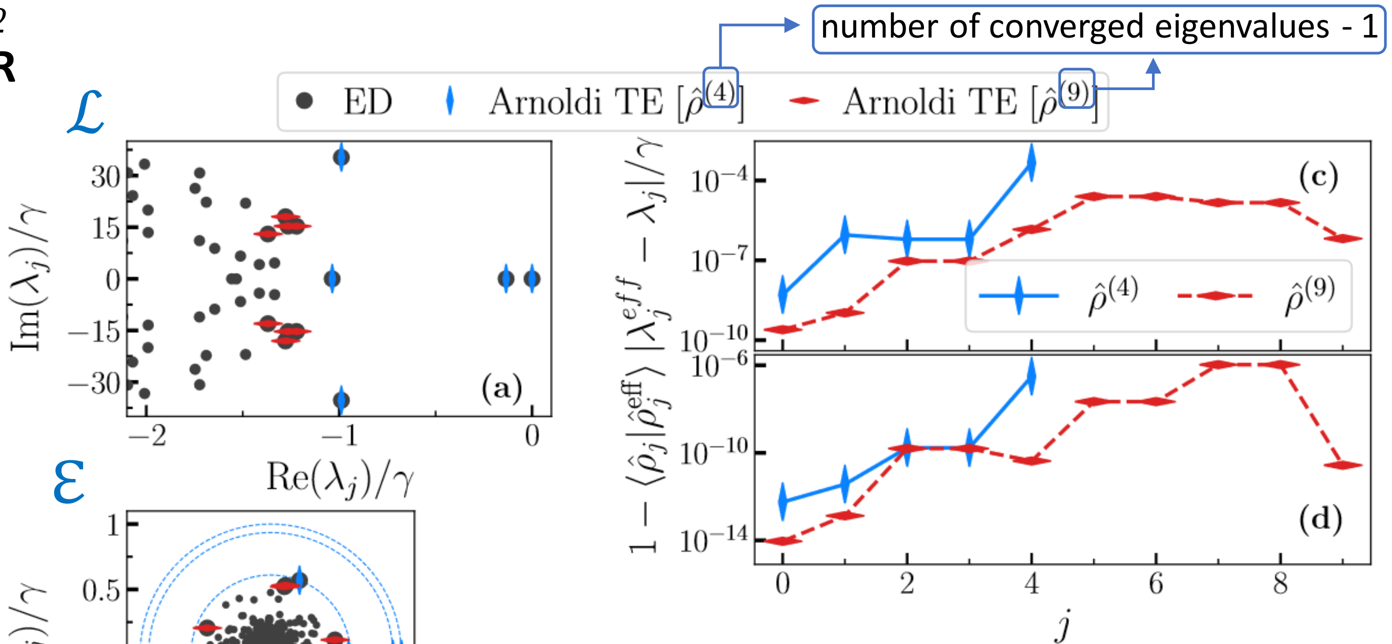
$U = 20\gamma$

$F = 4{,}5\gamma$

$n_{max} = 7$

$\Delta = 5\gamma$

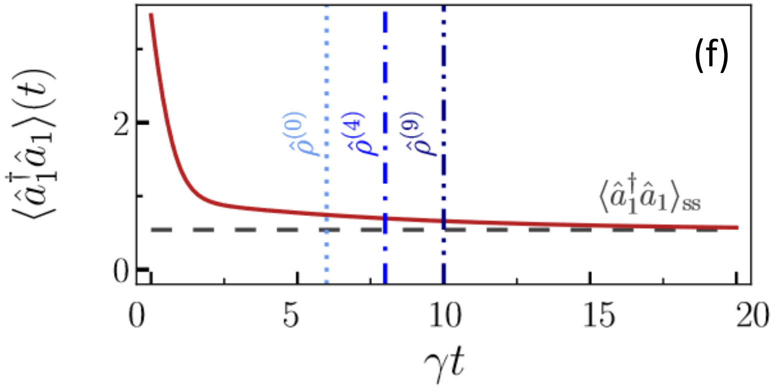
$J = 10\gamma$



number of converged eigenvalues - 1

(e)	ED	$\hat{\rho}^{(0)}$	$\hat{\rho}^{(4)}$	$\hat{\rho}^{(9)}$	$\hat{\rho}^{(49)}$
Computation Time	43 s	8.5 s	11.75 s	15 s	42 s
Evolution Time		$6/\gamma$	$8/\gamma$	$10/\gamma$	$14/\gamma$
$\dim(\mathcal{L}^{\mathrm{eff}})$	4096	120	160	200	280

- very small error on eigendecomposition
- faster and more precise



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FLOQUET SYSTEMS

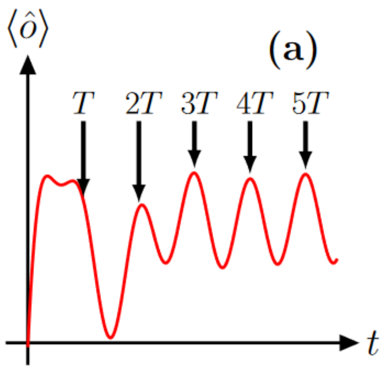
$$\partial_t \hat{\rho}(t) = \mathcal{L}(t)\,\hat{\rho}(t),$$

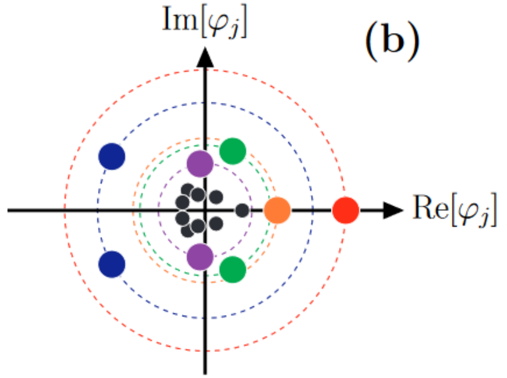
$$\mathcal{L}(t+T) = \mathcal{L}(t)$$

$$\hat{\rho}(T) = \mathcal{E}\hat{\rho}(0) \equiv \mathcal{F}\hat{\rho}(0)$$

$$= e^{\mathcal{L}^F T}\hat{\rho}(0)$$

“Floquet map”



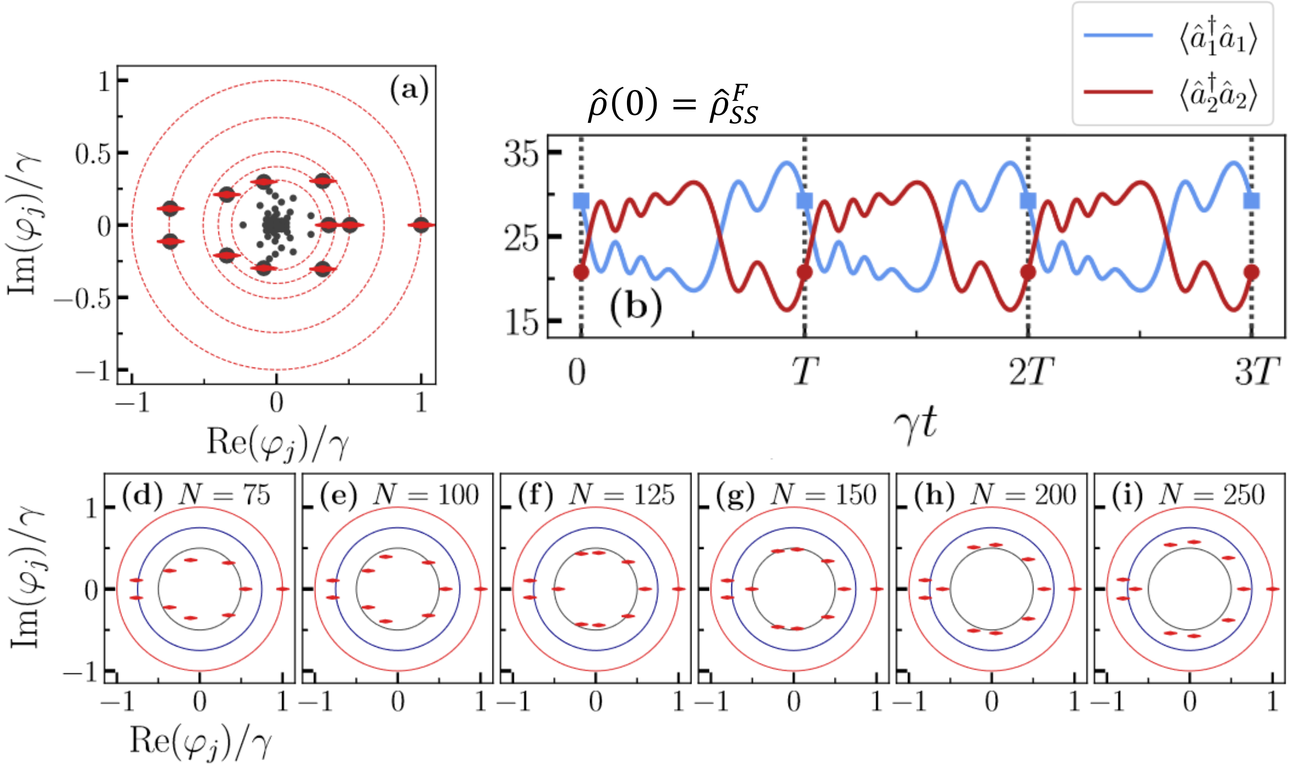


stroboscopic eigenspectrum

$$\mathcal{F}\hat{\rho}_j^F = \varphi_j^F\,\hat{\rho}_j^F$$

$$\Updownarrow$$

$$\mathcal{L}^F\hat{\rho}_j^F = \lambda_j^F\,\hat{\rho}_j^F$$



Example model: *M. Hartmann et al, New J. Phys. (2017)*

$$\hat{H}(t) = \frac{U}{2}\sum_{j=1}^{2}\hat{a}_j^\dagger\hat{a}_j^\dagger\hat{a}_j\hat{a}_j - J(\hat{a}_1^\dagger\hat{a}_2 + \hat{a}_2^\dagger\hat{a}_1) + f(t)(\hat{a}_2^\dagger\hat{a}_2 - \hat{a}_1^\dagger\hat{a}_1)$$

$$\hat{L} = (\hat{a}_1^\dagger + \hat{a}_2^\dagger)(\hat{a}_1 - \hat{a}_2) \qquad = f_0 + f_1\cos\omega t$$

Conclusions

- Method to efficiently determine the low-lying eigenvalues and eigenmatrices of the evolution operator/ Liouvillian;
- Retrieve these features through a (shorter) time evolution of the open system;
- Grants access to eigendecomposition of bigger system sizes.

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