Arnoldi-Lindblad time evolution: Faster-than-the-clock algorithm for the spectrum of (Floquet) open quantum systems

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\[
\begin{align*}
\partial_t \hat{\rho} &= -i \{ \hat{H}, \hat{\rho} \} + \sum_j \frac{\gamma_j}{2} \left( 2 \hat{L}_j \hat{\rho} \hat{L}_j^\dagger - \{ \hat{L}_j^\dagger \hat{L}_j, \hat{\rho} \} \right) = \mathcal{L}[\hat{\rho}] \\
\mathcal{L} \hat{\rho}_j &= \lambda_j \hat{\rho}_j
\end{align*}
\]

Lindblad master equation

Lindblad operator \textit{“dissipation operator”}

dissipation rate

closed system dynamics

coupling with environment

Liouvillian spectrum

\[ \text{Im}[\lambda_j] \]

\[ \text{Re}[\lambda_j] \]

Liouvillian eigendecomposition highly desired

exact diagonalisation often infeasible while time evolution is still accessible

main interest in “slow” states
e.g. the steady state

\[ \hat{\rho}(t) = e^{\mathcal{L}t} \hat{\rho}(0) = \sum_j c_j e^{\lambda_j t} \hat{\rho}_j \]

\[ \{ \hat{L}_j^\dagger \hat{L}_j, \hat{\rho} \} \]

\[ \hat{L}_j \hat{\rho} \hat{L}_j^\dagger \]

\[ \hat{L}_j \hat{\rho} \hat{L}_j^\dagger - \{ \hat{L}_j^\dagger \hat{L}_j, \hat{\rho} \} \]

Liouvillian superoperator

system

coupling

environment

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THE METHOD

Time evolution

\[ \dot{\hat{\rho}}(t) = \dot{\hat{\rho}}_{SS} + \sum_{j \geq 1} c_j e^{\lambda_j t} \dot{\hat{\rho}}_j \]

Spectrum transformation

\[ \mathcal{E} = e^{\mathcal{L}T} \]
\[ \dot{\hat{\rho}}(t) = \left[ e^{\mathcal{L}T} \right]^N \dot{\hat{\rho}}(0) = \mathcal{E}^N \dot{\hat{\rho}}(0) \]

Krylov and Arnoldi method

\[ K_n = \{ \dot{\hat{\rho}}(0), \dot{\hat{\rho}}(T), \dot{\hat{\rho}}(2T), \ldots, \dot{\hat{\rho}}(nT) \} = \{ \dot{\hat{\rho}}(0), \mathcal{E}\dot{\hat{\rho}}(0), \mathcal{E}^2\dot{\hat{\rho}}(0), \ldots, \mathcal{E}^n\dot{\hat{\rho}}(0) \} \]

Arnoldi iteration

Gram-Schmidt orthonormalisation

\[ \mathcal{E}_{n}^{\text{eff}} = \begin{bmatrix} e_{1,1} & e_{1,2} & e_{1,3} & \cdots & e_{1,n} \\ e_{2,1} & e_{2,2} & e_{2,3} & \cdots & e_{2,n} \\ 0 & e_{3,2} & e_{3,3} & \cdots & e_{3,n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & e_{n,n-1} & e_{n,n} \end{bmatrix} \]

check convergence

\[ \| \mathcal{E}_{n}^{\text{eff}} \beta_{j}^{\text{eff}} - \beta_{j}^{\text{eff}} \beta_{j}^{\text{eff}} \| < \tau \]

\[ \mathcal{L} \hat{\beta}_{j}^{\text{eff}} = \lambda_{j}^{\text{eff}} \hat{\beta}_{j}^{\text{eff}} \]
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\[ \hat{H} = \sum_{j=1}^{2} \left[ -\Delta \hat{a}_j^\dagger \hat{a}_j + \frac{U}{2} \hat{a}_j^\dagger \hat{a}_j^\dagger \hat{a}_j \hat{a}_j + F (\hat{a}_j^\dagger + \hat{a}_j) \right] - J (\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1) \]

\[ \partial_t \hat{\rho} = -i [\hat{H}, \hat{\rho}] + \frac{\gamma}{2} \sum_{j=1}^{2} (2 \hat{a}_j \hat{\rho} \hat{a}_j^\dagger - \{ \hat{a}_j^\dagger \hat{a}_j, \hat{\rho} \}) \]

parameters:

- \( U = 20 \gamma \)
- \( F = 4.5 \gamma \)
- \( n_{max} = 7 \)
- \( \Delta = 5 \gamma \)
- \( J = 10 \gamma \)

very small error on eigendecomposition

faster and more precise
Conclusions

- Method to efficiently determine the low-lying eigenvalues and eigenmatrices of the evolution operator/Liouvillean;
- Retrieve these features through a (shorter) time evolution of the open system;
- Grants access to eigendecomposition of bigger system sizes.