Spin squeezing in quantum simulators

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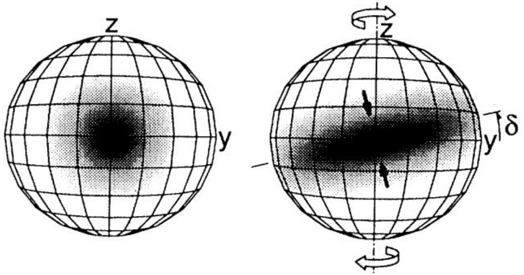
Spin squeezing: A form of multipartite entanglement, defined for the collective spin:

$$\xi_R^2 = \frac{N \min_\perp \mathrm{Var}[J^\perp]}{|\langle \mathbf{J} \rangle|^2}, \qquad \mathbf{J} = \sum_{i=1}^{N} \mathbf{S}_i$$



Coherent spin state vs spin-squeezed state
[Kitagawa&Ueda, PRA 1993]

Squeezed state ($\xi_R^2 < 1$):
- Reduced spin fluctuations in one direction orthogonal to $\langle \mathbf{J} \rangle$;
- Entangled state ($\xi_R^2 < 1/k$ implies entanglement depth $k+1$);
- Metrological gain in phase estimation via Ramsey interferometry.

Paradigmatic strategy: One-axis-twisting (OAT) model. Unitary dynamics induced by $H_{\mathrm{OAT}} = \chi (J^z)^2$ on the state $|\mathrm{CSS}\rangle_x = \otimes_{i=1}^N |\uparrow_x\rangle_i$ generates optimal squeezing scaling as $\xi_R^2 \propto 1/N^{2/3}$.

How to generate squeezing with realistic models for quantum simulators?

Summary

Quantum simulators give access to scalable spin squeezing.

1. For sufficiently long-range α-XX models (e.g. Rydberg atoms in two dimensions, with dipolar couplings), quench dynamics from $|\mathrm{CSS}\rangle_x$ (polarized product state) yields

$$\xi_R^2 \propto 1/N^{2/3}, \qquad t \propto N^{1/3}.$$

2. For systems that spontaneous break a continuous symmetry (e.g. short-range XXZ model in optical-lattice Mott insulators), quasi-adiabatic preparation towards small field yields

$$\xi_R^2 \propto 1/N^{1/2}, \qquad t \propto N.$$

References:
- Comparin, Mezzacapo, Roscilde, “Universal spin squeezing from the tower of states of U(1)-symmetric spin Hamiltonians”, arXiv (2021).
- Roscilde, Mezzacapo, Comparin, “Spin squeezing from bilinear spin-spin interactions: two simple theorems”, Phys. Rev. A 104, L040601 (2021).
- Comparin, Mezzacapo, Robert-de-Saint-Vincent, Vernac, Laburthe-Tolra, Roscilde, “Scalable spin squeezing from spontaneous breaking of a continuous symmetry” (in preparation).

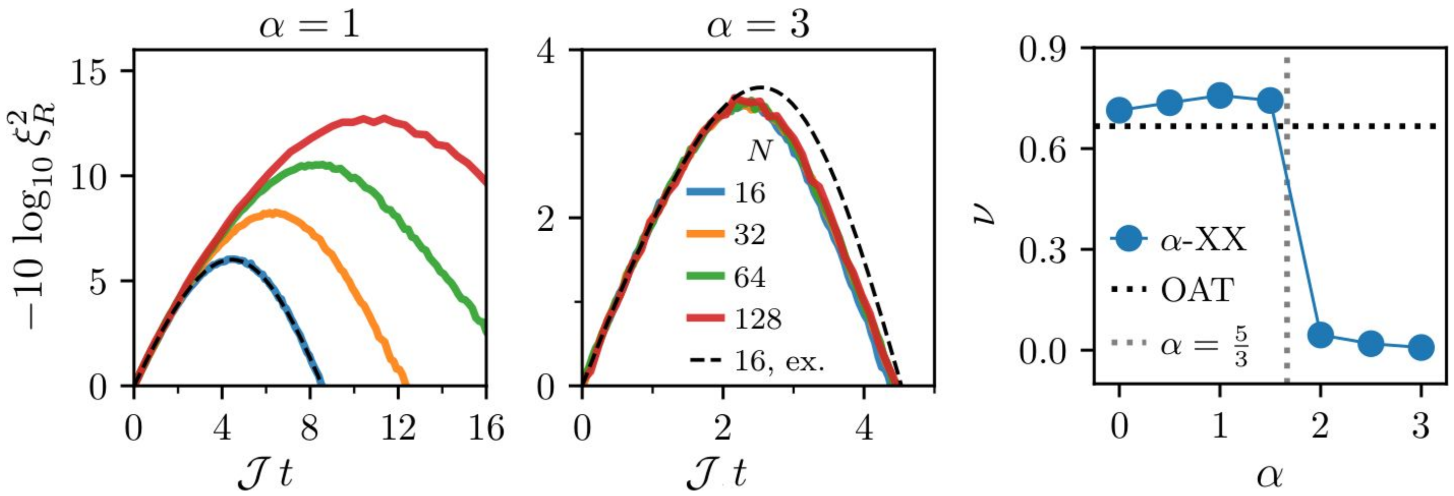
Quench dynamics for α-XX models

$$H = -\sum_{i<j} \frac{\mathcal{J}}{|\mathbf{r}_i - \mathbf{r}_j|^\alpha} \left(S_i^x S_j^x + S_i^y S_j^y \right)$$

Evolve an initially x-polarized state with α-XX Hamiltonian
(dynamics simulated via time-dependent Variational Monte Carlo and Jastrow Ansatz).

Optimal-squeezing at finite time, scaling as $\xi_R^2 \propto 1/N^\nu$

(1) OAT model at α=0; (2) OAT scaling up to finite α; (3) no scaling for shorter-ranged couplings.



Comparin, Mezzacapo, Roscilde, “Universal spin squeezing from the tower of states of U(1)-symmetric spin Hamiltonians”, arXiv 2021.

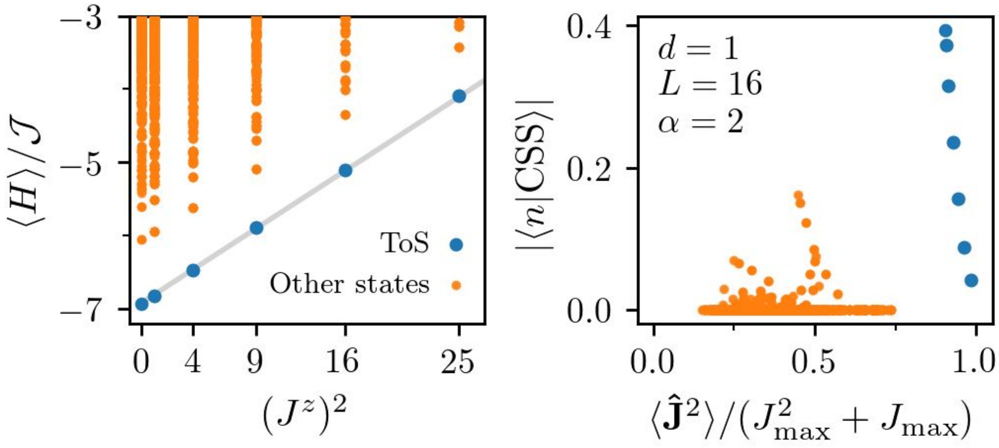
Anderson Tower of States (ToS)

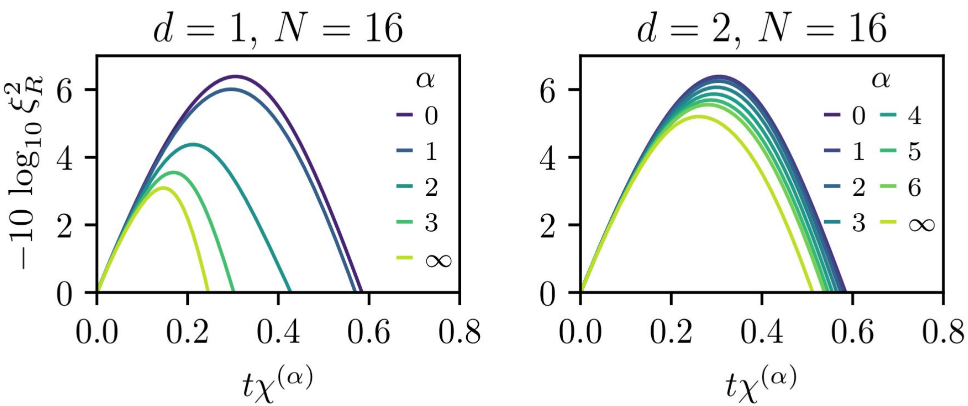
A set of energy eigenstates with

$$E \propto (J^z)^2, \quad E \propto 1/N.$$

Signature of symmetry breaking in finite-size spectra [Anderson, PR 1952].

Also present for α-XX models, when breaking U(1) symmetry.





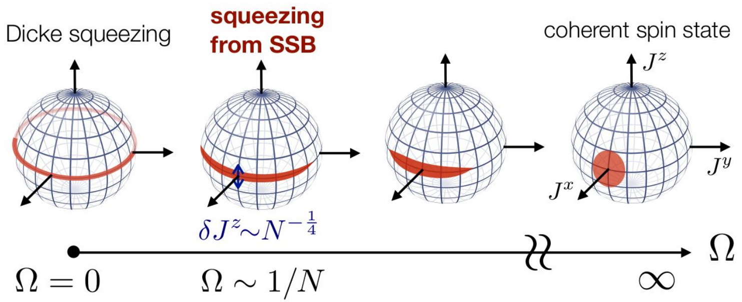
Anomalously large total spin + large overlap with initial (CSS) state: quantum scars.

Short-time dynamics remains in sector of max total spin, where it maps onto an OAT model:

$$H \simeq \chi^{(\alpha)} (J^z)^2$$

Consequence: robustness of short-time squeezing generation.

Squeezing from (quasi)-adiabatic dynamics for symmetry-breaking models

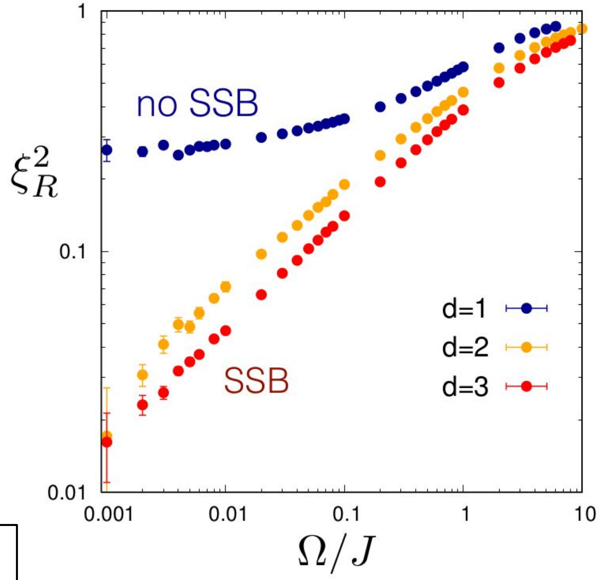


Spontaneous symmetry breaking (SSB) of continuous symmetry [e.g. U(1) for the XXZ model] → Anderson Tower of State (ToS), with energies scaling as $1/N$.

Strategy: add a strong symmetry-breaking field Ω, and decrease it adiabatically towards $\Omega \sim 1/N$:

1. Energy gap at small fields is $\propto \Omega^{1/2}$.
2. SSB: Microscopic field ($\propto 1/N$) mixes the ToS and induces macroscopic magnetization.
3. Fluctuations of J^z (symmetry generator) are suppressed, and they vanish for $\Omega = 0$

→ spin squeezing, confirmed by ground state calculations



QMC for n.n. XXZ model (see next slide)

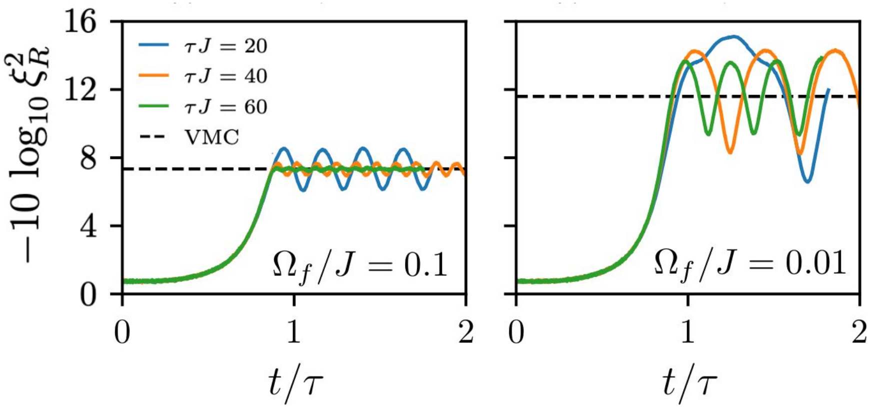
Comparin, Mezzacapo, Robert-de-Saint-Vincent, Vernac, Laburthe-Tolra, Roscilde, "Scalable spin squeezing from spontaneous breaking of a continuous symmetry" (in preparation)

Short-range XXZ model in two dimensions

Ground state Quantum Monte Carlo for n.n. XXZ model:

$$H = -J \sum_{\langle i,j \rangle} (S_i^x S_j^x + S_i^y S_y^y - \Delta S_i^z S_j^z) - \Omega \sum_i S_i^x$$

for $\Delta = 1$ (this slide) and $\Delta = 0$ (not shown).



1. No need for strict adiabaticity (see variational simulation of non-adiabatic ramp dynamics).
2. Minimal spin uncertainty (i.e. saturated Heisenberg-Robertson inequality)
   → rotation of the average collective spin around y (Ramsey interferometry) is the optimal measurement.