Optimal control of the quantum state of a BEC in an optical lattice

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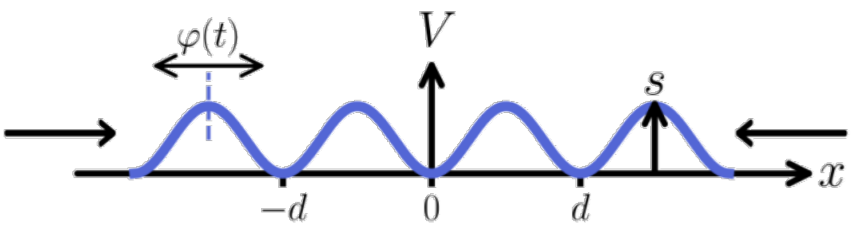


$$V(x,t) = \frac{sE_L}{2}[1-\cos(k_L x + \varphi(t))]$$

$$k_L = \frac{2\pi}{d},\ E_L = \frac{\hbar^2 k_L^2}{2m}$$

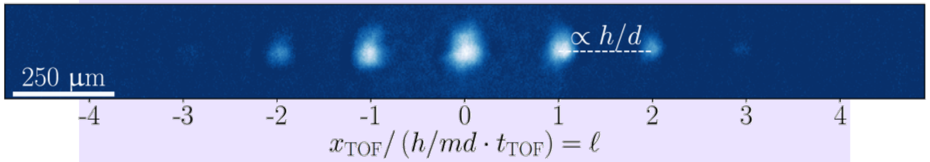
In the $q=0$ subspace, wavefunction :

$$|\psi\rangle = \sum_{\ell\in\mathbb{Z}} c_\ell\,|\chi_\ell\rangle$$

(expansion on plane waves $\chi_\ell(x) \propto e^{i\ell k_L x}$)

Imaging : momentum distribution

atoms imaged after time-of-flight (TOF)



Initial wavefunction = ground state quasimomentum $q=0$

Schrödinger equation :

$$i\dot{c}_\ell = \ell^2 c_\ell - \frac{s}{4}\left(e^{i\varphi(t)}c_{\ell-1} + e^{-i\varphi(t)}c_{\ell+1}\right) \Leftrightarrow i\dot{C} = \mathcal{M}\left(\varphi(t)\right)\times C$$

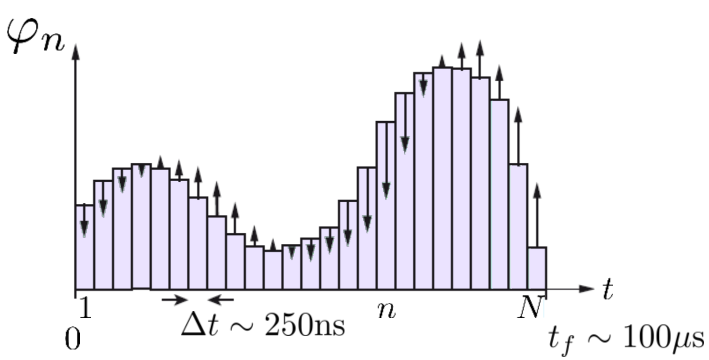
We engineer the final state $C(t_f)$ using the control parameter $\varphi(t)$

Derivation of optimal control

❖ Choose a target state C_T,
a figure of merit $\mathcal{F}$ (fidelity)

$$\text{e.g.}\quad \mathcal{F} = |C_T^\dagger C(t_f)|^2$$

❖ Optimize a discretized ramp φ_n:



Iterative optimization (gradient ascent)

For control $\{\varphi_n^{(k)}\}$:

- compute $C(t)$, and the *adjoint* $D(t)$:

$$D(t_f) = \frac{\partial \mathcal{F}}{\partial C^\dagger(t_f)} \qquad i\dot{D} = \mathcal{M}\left(\varphi(t)\right) \times D$$

- build *Pontryagin's Hamiltonian*

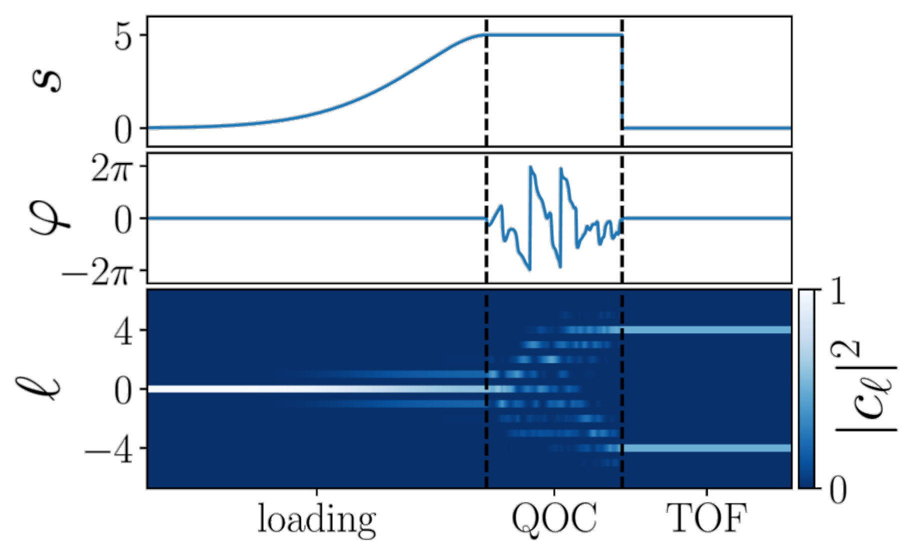
$$H_\mathrm{P} = \mathrm{Re}\left(D^\dagger \dot{C}\right)$$

- Apply correction

$$\varphi_n^{(k)} \to \varphi_n^{(k+1)} = \varphi_n^{(k)} + \epsilon \frac{\partial H_\mathrm{P}}{\partial \varphi_n^{(k)}}$$

Experimental sequence :

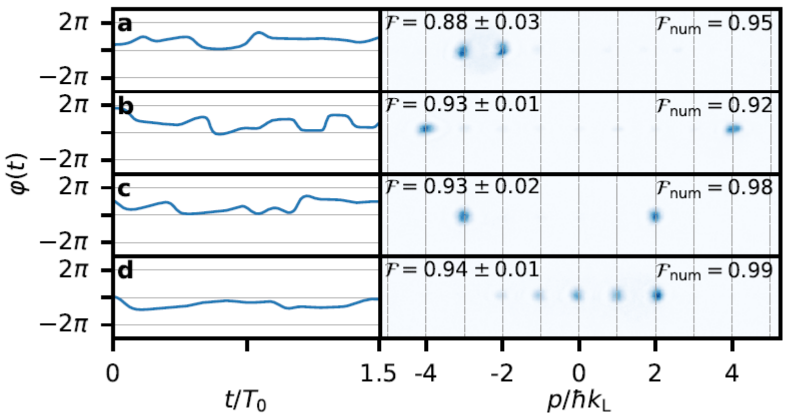
- precise lattice depth calibration*
- adiabatic lattice loading
- optimal phase control
- time-of-flight, imaging

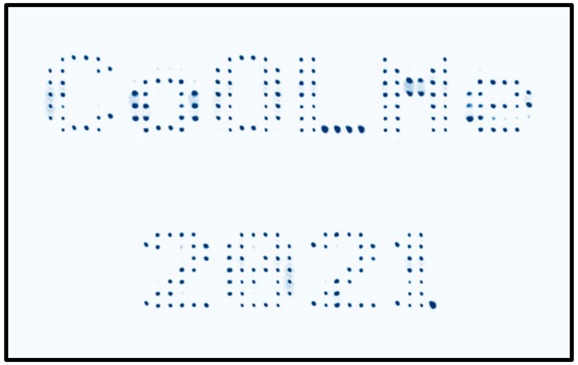


*C. Cabrera-Gutiérrez et al., *Phys. Rev. A* **97**, 043617 (2018)

Experimental results

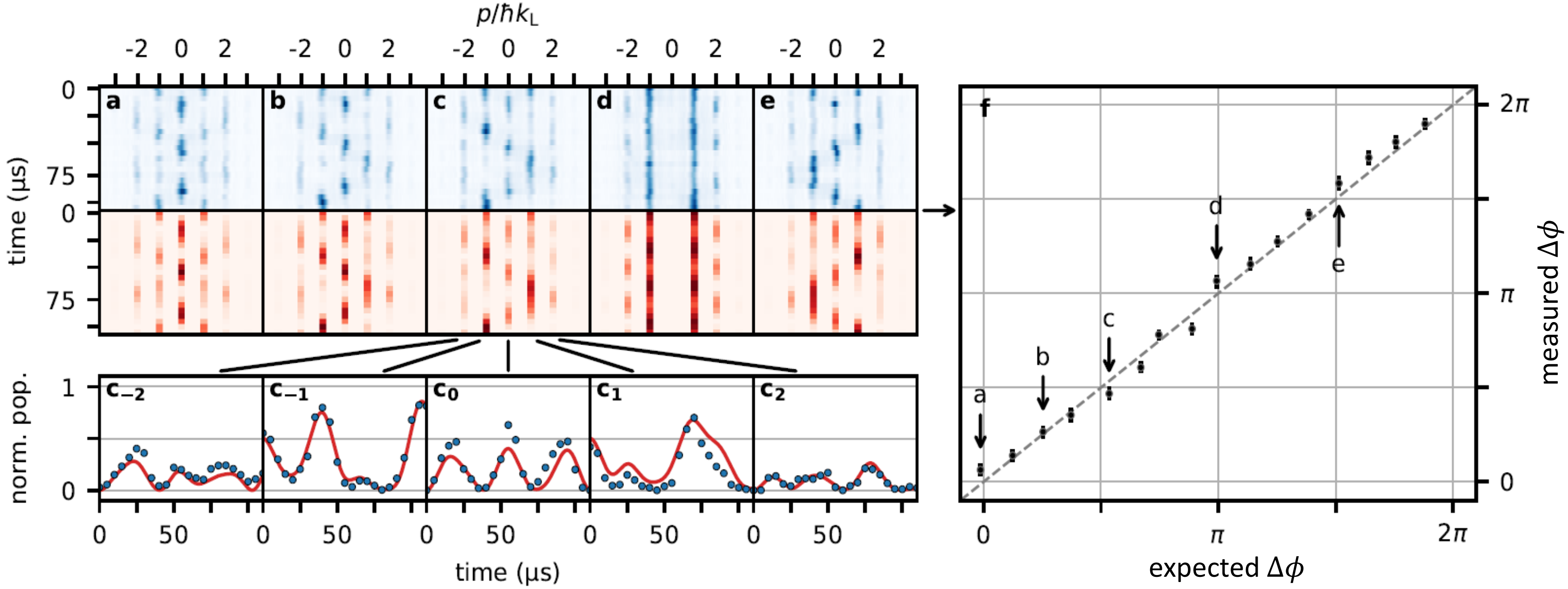
- States with equal weights/arbitrary weights on chosen momentum peaks





- State $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\chi_1\rangle + e^{i\Delta\phi}|\chi_{-1}\rangle\right)$ with given relative phase $\Delta\phi$,

Phase measurement from subsequent evolution



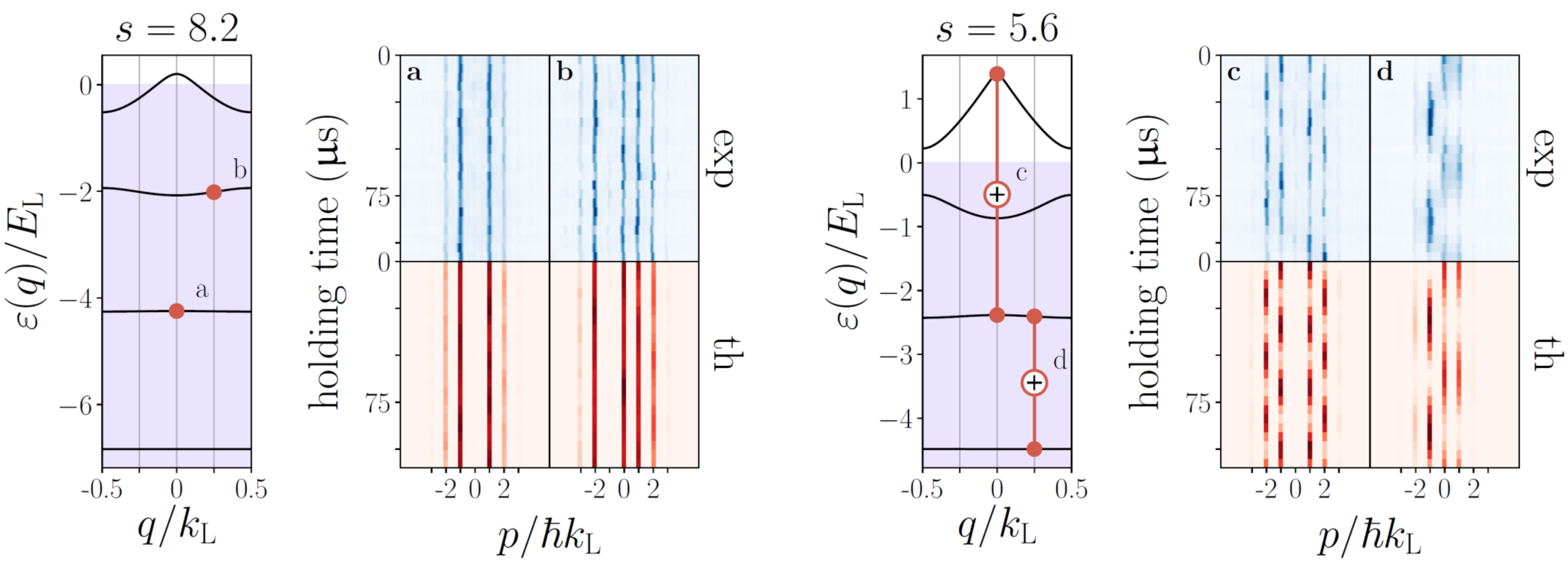
Preparation of eigenstates

At quasimomentum $q = \tilde{q}k_L$, the n^{th} Bloch function reads

$$|\psi_{n,q}\rangle = \sum_{\ell\in\mathbb{Z}} c_\ell^{(n,q)} |\chi_{\ell+\tilde{q}}\rangle$$

With $c_\ell^{(n,q)}$ solutions of the stationary Schrödinger equation

❖ We prepare eigenstates and superpositions thereof



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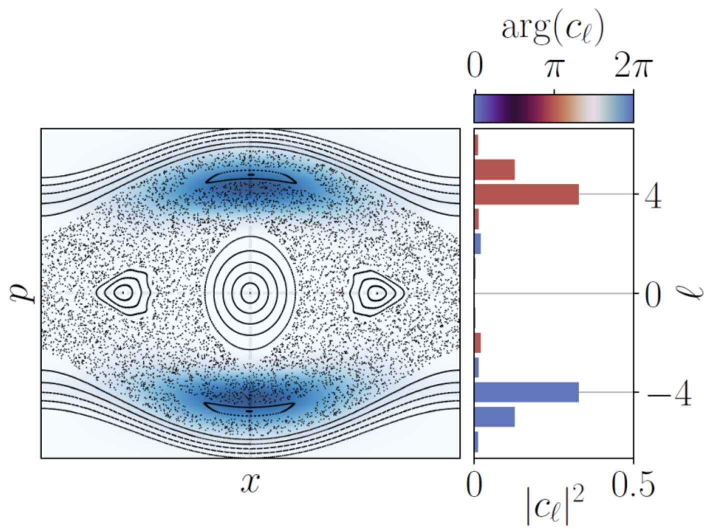
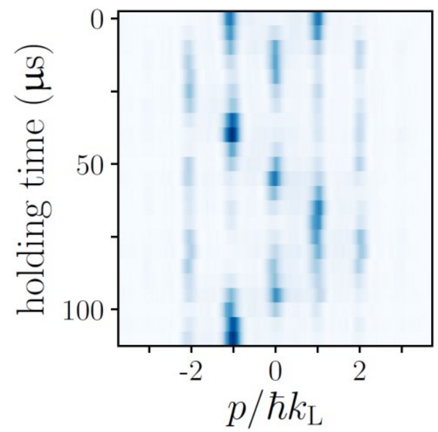
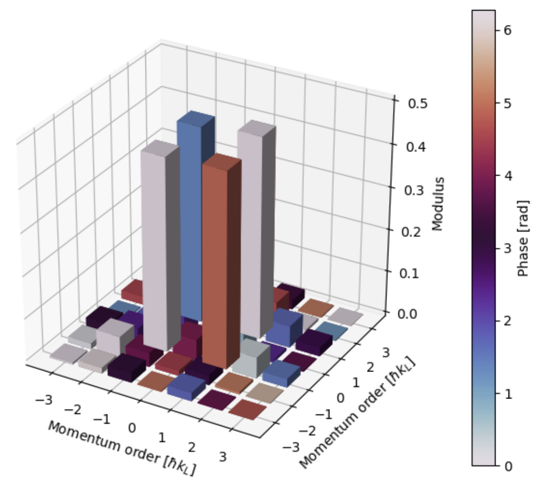
Perspectives

- Preparation of states in a phase space of interest, e.g. Floquet mixed dynamical system (within a lattice cell)
- Full estimation of the prepared state: subsequent dynamics provide a ensemble of measurements for maximum likelihood reconstruction





Target $\frac{1}{\sqrt{2}}(|\chi_1\rangle + i|\chi_{-1}\rangle)$



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