Imaging the interface of a qubit and its quantum-many-body environment

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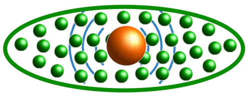
Research Status

- The work is union of two disciplines in ultra cold atomic physics: Rydberg physics and Bose-Einstein condensation.

Rydberg atom in BEC

- Rydberg atom is an excited atom with one or more electrons that have a very high principal quantum number.



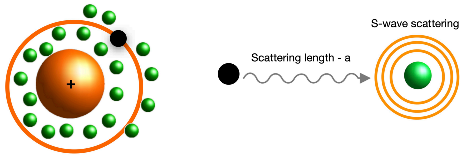
Rydberg atoms were first excited in BEC at nK temperature in 2004.

- Creation of Rydberg atoms in BEC creates phonons.
- The field operator of the BEC then can be written as

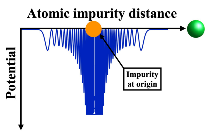
$$\hat{\Psi}_g(\mathbf{x}) = \phi_0(\mathbf{x}) + \sum_{\mathbf{q}} \left(u_{\mathbf{q}}(\mathbf{x})\, \hat{b}_{\mathbf{q}} - v^*_{\mathbf{q}}(\mathbf{x})\, \hat{b}^\dagger_{\mathbf{q}} \right)$$

Interaction of Rydberg-ground state atom

- The Rydberg electron wave function extends through a large area of the background BEC and the dominant interactions are elastic scattering.

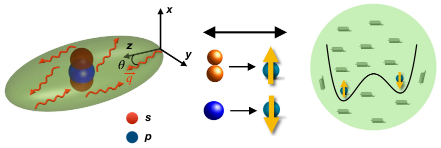


- The interaction potential is mainly due to scattering is given by [3]



$$V(R) = \underbrace{\frac{2\pi\hbar^2 a}{m_e}}_{g_0} |\psi(R)|^2$$

Rydberg Impurity in BEC as Spin Boson Model

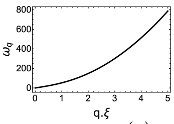


$$\hat{H}_{syst} = \frac{\Omega_{mw}}{2}\hat{\sigma}_x + \frac{\Delta E}{2}\hat{\sigma}_z$$

$\Omega_{mw} \rightarrow$ Microwave coupling frequency

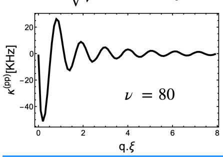
$\Delta E = E_p - E_s$

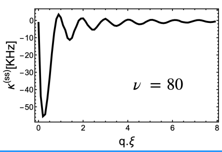
$$\hat{H}_{env} = \sum_{\mathbf{q}} \hbar\omega_{\mathbf{q}}\, \hat{b}^\dagger_{\mathbf{q}} \hat{b}_{\mathbf{q}}$$



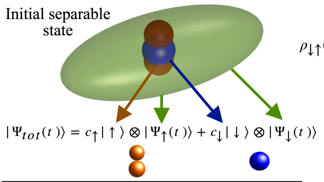
$$\hat{H}_{int} = \sum_{\mathbf{q}} \frac{\Delta\kappa_{\mathbf{q}}}{2}\left(\hat{b}_{\mathbf{q}} + \hat{b}^\dagger_{\mathbf{q}}\right)\hat{\sigma}_z \implies \Delta\kappa_{\mathbf{q}} = \kappa^{(p)}_{\mathbf{q}} - \kappa^{(s)}_{\mathbf{q}}$$

$$\kappa^{(\alpha)}_{\mathbf{q}} = \frac{g_0\sqrt{\rho}}{\sqrt{V}}(\bar{u}_{\mathbf{q}} - \bar{v}_{\mathbf{q}}) \int d^3\mathbf{x}\, |\psi^{(\alpha)}(\mathbf{x})|^2 e^{i\,\mathbf{q}\cdot\mathbf{x}} \rightarrow \alpha = s \text{ or } p$$





Decoherence time



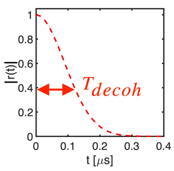
Spin coherence

$$\rho_{\downarrow\uparrow}(t) = c^*_\downarrow c_\uparrow \langle \Psi_\uparrow(t) | \Psi_\downarrow(t) \rangle$$

SBM **GPE**

$$|\Psi_{tot}(t)\rangle = c_\uparrow |\uparrow\rangle \otimes |\Psi_\uparrow(t)\rangle + c_\downarrow |\downarrow\rangle \otimes |\Psi_\downarrow(t)\rangle$$

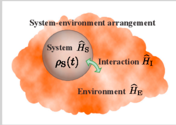
$$r(t) = \langle \Psi_\uparrow(t) | \Psi_\downarrow(t) \rangle = exp\left[-\sum_{\mathbf{q}} \frac{\Delta\kappa^2_{\mathbf{q}}}{2\hbar} t^2 \right]$$



n=40	n=120
$T_{decoh} \approx 20\ ns$	$T_{decoh} \approx 0.9\ \mu s$

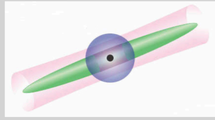
Theoretical Background

Open quantum systems



Hong-Bin Chen et.al, Phys. Rev. Lett. **120**, (2018) 030403

System and Environment



K. S. Kleinbach et. al, Phys. Rev. Lett **120**. (2018) 193401

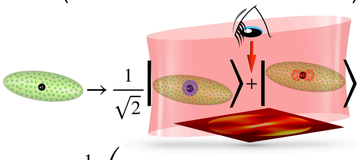
Impurity in BEC

- We consider two level impurity atom as a system in an environment of BEC and study the decoherence and Non-Markovian features.

Mean field approach

- The dynamics of BEC is given by GPE.

$$i\hbar\frac{\partial}{\partial t}\phi_\alpha(\mathbf{r}) = \left(-\frac{\hbar^2}{2m}\nabla^2 + U_0|\phi_\alpha(\mathbf{r})|^2 + g_0|\psi^\alpha(\mathbf{r})|^2 \right)\phi_\alpha(\mathbf{r})$$



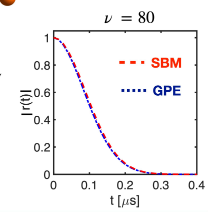
$$|\Psi(t)\rangle = \frac{1}{\sqrt{2}}\Big(|p\rangle \otimes |\phi_p(t)\rangle + |s\rangle \otimes |\phi_s(t)\rangle \Big)$$

Many body state — Boson mean field wave function in presence of p — Boson mean field wave function in presence of s

$$\phi_\alpha(x,t) = \langle \hat{\Psi}(x,t) \rangle$$

$$|r_{GPE}(t)| = \left[\frac{\int d^3\mathbf{r}\, \phi^*_p(\mathbf{r},t)\phi_s(\mathbf{r},t)}{N} \right]^N$$

- SBM and GPE results match →



Signature of system bath entanglement

- Total atom density in many-body state $|\Psi_\alpha(t)\rangle$

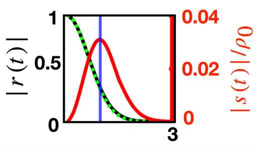
$$\varrho(\mathbf{r}) = N \int d^3\mathbf{x}_2 \ldots \int d^3\mathbf{x}_N \left| \Psi(\mathbf{r}_2, \mathbf{x}_2, \ldots, d^3\mathbf{x}_N) \right|^2$$

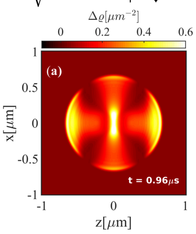
Coherent mixture

$$|\Psi_{bg}\rangle = A\left[|\Psi_\uparrow\rangle + |\Psi_\downarrow\rangle \right] \quad \text{where} \quad A = \frac{1}{\sqrt{2(1 + Re[\langle \Psi_\uparrow | \Psi_\downarrow \rangle])}}$$

Classical mixture $\varrho_{inc} = \frac{\varrho_\uparrow + \varrho_\downarrow}{2}$

$$\Delta\varrho = \varrho_{coh} - \varrho_{inc}$$





NMQSD

- Non-Markovian Quantum State Diffusion is a method to study Non-Markovian open quantum systems.
- Considering general case of an open quantum system:

$$\hat{H}_{tot} = \hat{H}_{sys} + \sum_\lambda \omega_\lambda \hat{a}^\dagger_\lambda \hat{a}_\lambda + \sum_\lambda \left(g^*_\lambda \hat{L} \otimes \hat{a}^\dagger_\lambda + g_\lambda \hat{L}^\dagger \otimes \hat{a}_\lambda \right)$$

$$\hat{a}_\lambda \rightarrow \hat{b}_q \quad ; \quad g_\lambda \rightarrow \Delta\kappa_q \quad ; \quad \hat{L} \rightarrow \hat{\sigma}_z$$

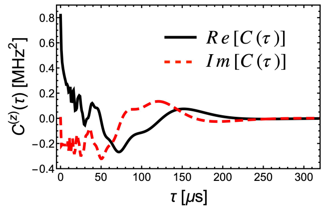
- Initial state: $|\Psi\rangle = |\psi_0\rangle \otimes |0\rangle$
- Reduced density matrix is obtained from ensemble average over trajectories of pure states $|\psi_t(z^*)\rangle$.
- Interaction between system and environment is incorporated as a Gaussian white noise z.
- Cross correlation of the noise.

$$M_z[z^*_n(t) z_n(s)] = C_n(t,s) = \sum_i \left[\frac{g_i^2}{2m_i\omega_i} \Big(\cos(\omega_i(t-s)) - i\,\sin(\omega_i(t-s)) \Big) \right]$$

- NMQSD:

$$\partial_t |\psi(t,z)\rangle = -i\hat{H}_{sys}|\psi(t,z)\rangle + \sum_n L_n z^*_n(t)|\psi(t,z)\rangle - \sum_n L^\dagger_n \int_0^t ds\, \alpha_n(t-s) \frac{\delta}{\delta z^*_n(s)}|\psi(t,z)\rangle$$

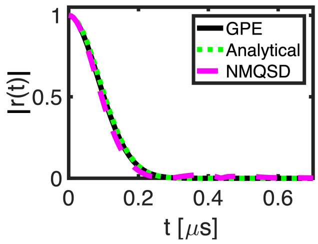
- The reduced density operator $\rho(t) = M_Z\Big[|\psi(t,z)\rangle\langle\psi(t,z)| \Big]$
- NMQSD is solved using a numerical method called Hierarchy of Pure States (HOPS) [5].
- Correlation function:



- Correlation function is fitted using sum of exponentials [5].

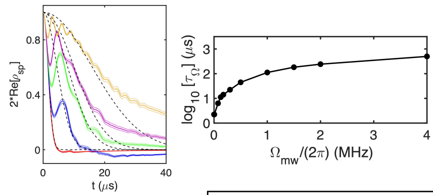
$$C(\tau) = \sum_i g_i\, e^{-\omega_i \tau} \quad \text{where} \quad \omega_j = \gamma_j + i\Omega_j$$

- For the above bath correlation function, we solve NMQSD using HOPS.
- Comparison of decoherence time from all three methods [1].



Non-Markovian features

- For $\Omega_{mw} \neq 0$



- Quantifying Non-Markovinity measure [6]

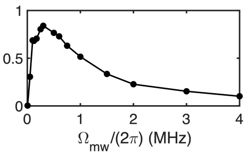
Trace distance between s state and p state density matrices

$$D(S,P) = \frac{1}{2} Tr[S - P]$$

- Non-Markovian condition

$$\sigma(t, P(0), Q(0)) = \frac{d}{dt} D(S(t), Q(t)) > 0$$

$$N_{S,P} = \int_{\sigma > 0} dt\ \sigma(t, S(0), P(0))$$



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